The Mathematician and Spaceflight

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The art of spaceflight, that is, the design and launching of vehicles into desired orbits around the earth or other celestial body, is now less than eight years old. For hundreds of years before that, however, mathematicians and astronomers had successfully described and predicted the orbits of natural celestial bodies by means of mathematics. Small wonder, then, that mathematicians have always been essential contributors to spaceflight design and operations.

Any mathematics thus used directly for solving problems arising in science or technology is called Applied Mathematics. The applied mathematician must therefore have an interest in making sufficient acquaintance with the definitions and patterns of thought of whatever field of science or technology in which his current problem may lie -- as well as _expertise_ in whatever branch of mathematics seems likely to yield a solution to that problem. Discernment of just what is the best mathematics to apply to any problem is part of the problem itself, and the art of such discernment is a key technique for successful practice of applied mathematics. Indeed, the greatest applied mathematicians may even go so far as to _invent_ new mathematics to deal with the problem at hand. Thus in order to solve the problem of describing the motion of bodies acted upon by various forces, Isaac Newton invented the Calculus, and to discuss averages and random errors as affecting astronomical and geodetic observations Gauss invented much of Probability and Statistics, including the famous Gaussian Curve.

Any problem in applied mathematics may directly entail not only various branches of mathematics but also some of the many fields of science and engineering, indicated by the subject matter of the problem to which the appropriate mathematics is to be applied. So in order to approach the question: "What mathematics is needed for solving problems of spaceflight?" we must first consider the question "What fields of science and technology are involved in spaceflight?" and then look about in these fields at problems amenable to possible mathematical attack.

The most basic and elemental requirement for artificial spaceflight, a requirement which distinguishes it from airflight and all other flight before 1957, is that the speed of the spacecraft must be great enough so that it will travel freely at least once around the earth without any additional propelling force and without falling to the ground. Since, as everyone knows, the faster a ball is thrown the further it will travel, the general conclusion seems obvious that the speed required for spaceflight must be very high -- but exactly how high? It is obvious also that to attempt to answer this question by simply experimenting with high speed projectiles and rockets until one was found which could circumnavigate the earth would be prohibitively expensive and dangerous. Thus, as Goethe had Mephistopheles advise Faust on a very different problem, "We cannot storm the town, in short, so must to stratagem resort." And the stratagem for this impasse, as for many others in spaceflight, is mathematics. Indeed, this particular problem was solved nearly three hundred years ago by Isaac Newton as part of his general problem of motion mentioned above, and it turns out that the required speed is just under 5 miles per second. Nowadays, we call the related field of physical science Mechanics, and the applicable branches of mathematics are the Differential and Integral Calculus, and Differential Equations. More specific scientific areas result when considering whether such a spacecraft travelling around the earth is or is not affected by forces other than gravitation, such as air resistance. If there is gravitation alone (the simpler case) the scientific field is called celestial mechanics, and all the mathematical theories of orbital motion, worked out by Newton and his followers for natural celestial objects, may be applied to the motion of an artificial spacecraft to determine or predict its path in space. But, if the spacecraft motion is affected in addition by air friction, the science of aerodynamics, also founded by Newton, must be included to develop a complete mathematical form of the spacecraft's differential equation of motion. Finally, since air friction depends on air density, we must look to the science of geophysics to supply a model of the earth's atmosphere so that the aerodynamic force in the equation of motion will be a correct and workable function of the height above the earth of the spacecraft. It turns out that, from theoretical considerations of gravity force and gas pressure, air density falls off exponentially with height, so the required function is transcendental. The differential equation of motion is a mathematical statement of Newton's second law of motiom, in which the acceleration is expressed as the second derivative of position with respect to time, and the mass and all forces involved are functions of position and time.

Another mathematical characteristic of spaceflight which distinguishes it from all other previous artificial travel is that, on a large scale, it is fully <u>three-dimensional</u>. All other movement planned by man is, relatively, just a creeping along or near the surface of the earth, that is, essentially travel with freedom in two dimensions only. Indeed, except for exact considerations of navigation on relatively long ocean and air trips, the earth's surface could even be considered as flat -- just as it always was before Columbus. Even air travel usually follows the earth's surface within ten miles -- much less than deviations from the true spherical shape of much of the earth's surface. Only with spaceflight did man really achieve three degrees of freedom in artificial motion.

Thus since the positions and motions of spacecraft must be studied and described in three dimensions, the mathematician so engaged must be familiar and expert with Solid Geometry in all its forms and ramifications. Of course, the Cartesian analytic geometry is most important here as being the usual mathematical formulation of analytic mechanics for studying motion. However, the more general conceptions of the Euclidean approach to Solid Geometry are often especially useful in large-scale space problems for the purpose of grasping the overall situation before tearing it into unrecognizable details by algebraic and numerical analyses. Study of conic sections in secondary school geometry is a most valuable basis for mathematical work on problems of spaceflight orbits about spheroidal bodies, which are fundamentally conic section curves and surfaces of revolution, respectively.

A particular field of basic importance for any study of outer space -- or even for navigating far reaches of the oceans and air of the earth -- is the geometry of the sphere. The sky,although actually immaterial, appears to us as a blue sphere with the observer at its center. Hence, for precise discussion of relative positions and motions of heavenly bodies, mathematical astronomy has, for many centuries, found spherical geometry and trigonometry most useful -- indeed, it is likely that these branches of mathematics were invented largely for this purpose. Landmarks during spaceflight can be nothing but the apparent directions of celestial bodies. We are fortunate, therefore, that the accumulated fund of knowledge and practice in spherical astronomy stands ready, after but little adaptation, for application to mathematical problems of spaceflight.

So far we have been thinking only of the translational motion of a spacecraft, that is, the velocity and acceleration of its center of gravity point. But the rotational motion of the body of the spacecraft is just as important, not only to plan for and to control the direction of thrust of the rockets which accelerate it, but also to make sure that directional instruments aboard are pointing where we want them to. Thus the TV camera aboard must be pointing to interesting parts of, say, the Moon, Venus, or Mars, while at the same time its directional antenna is pointing toward the earth, so that we may get the valuable pictures such as those which were returned to us from Ranger VII. . . To arrange this, a photo-electric sensor is built into the spacecraft so that its direction of sensitivity makes just the correct angle with camera or rocket, so that, if it is pointed toward a particular celestial object, camera and rocket will be correctly oriented. You may recall that the spacecraft Mariner IV, now headed for Mars, was rotated by small tangential rockets attached to it until its photo-sensor pointed toward the star Canopus. To design this situation required Spherical Trigonometry and Astronomy to calculate the spatial orientation of Canopus with respect to the required spacecraft orbit. This orbit, in turn, had to be planned using the Differential Equations of Celestial Mechanics, while the accelerations given to the spaceprobe by its attached rockets could be computed only by using the mathematical theory of rocket mechanics.

Although rocket performance is usually checked by static tests on the ground, the performance is also designed and predicted mathematically using chemical thermo-dynamics theory of the burning fuel, gas dynamics theory of the effect of engine throat shape, etc. Like any other structure, of course, the rocket as a whole must also be designed, using the mathematics of Analytic Mechanics, to make sure that its mechanical strength is sufficient to withstand all stresses imposed by forces acting on it, but, since for spaceflight the condition of <u>minimum</u> necessary weight is so crucial, the mathematical theory used here must be unusually careful and precise.

The rotational motion of a craft in the zero-gravity of outer space has a condition of complete freedom which is entirely beyond our experience on the ground, where gravity always produces interfering torques. As a result of this freedom, the axial rotation of a satellite, after many days and weeks in orbit around the earth, shows surprisingly large changes of speed and direction due to continued application of torques so small as to be completely negligible in the mechanics of an earth-bound rotor. An example of such effect is the observed steady slowing up of many satellites' rotation of such a nature that the rate of decline of the logarithm of rotation speed is constant with time. Thus, the time for decrease of rotation rate from, say, 4 per second to 2 per second is observed to be the same as the time for further decrease from 2 per second to $\sqrt{2} = 1.4$ per second, etc. Even before there were any artificial satellites, mathematical discussions had predicted such an effect as due to an electromagnetic torque induced by the magnetic field of the earth on a rotating satellite just as the rotor of an induction motor is turned by the rotating magnetic field of the stator. In order to check this theory against the rotational history of actual satellites, it was necessary to apply a combination of the mathematics of electricity and magnetism, of the earth's magnetic field, as well as of the orbit of the satellite in question.

Although the atmosphere several hundred miles up is extremely rare, its retarding force on spacecraft has an appreciable effect, a complicating factor which must be included in the calculation of orbits and rotation. This force may become very important at lower levels when, as on reentry of the Project Mercury capsules, it was necessary to compute the exact position of landing in order to salvage the astronaut. For this occasion, as for planned entry into atmospheres of other planets such as Mars, the frictional heating must also be computed to make sure of a safe design.

The necessity of continually following all objects which have been put into orbit has required the regular employment of a number of powerful electronic digital computers to predict future motion from observed positions. Large numbers of mathematicians are active here at devising mathematical programs in the fields suggested above, and also to check on the performance and to interpret the numerical results produced by the machines. For any such massive computation operation the field of mathematics known as Numerical Analysis is necessary for efficient achievement of meaningful results. By means of Computer Science it may also be possible to devise a useful adaptation of the available machine to the special mathematical operations required. One problem of spaceflight usually solved by such numerical methods is that of Optimum Trajectory Programming, that is, planning the amounts and directions of launching rocket thrust so as to achieve the required orbit for the spacecraft with a minimum of weight and consumption of fuel. In such a discussion all phases of rocket engineering as well as orbital science are potentially involved. A related question is that of Reliability, a complicated problem in Statistics involving combinations of the probabilities of failure of the many hundreds of thousands of parts of the rocket and its auxiliary instruments. Such mathematical investigations made in advance of actual spaceflight may possibly save millions of dollars, as well as, if the spacecraft be manned, one or more lives.

There are many other examples of possible application of various branches of mathematics to the fields of science and technology which are more or less essential to spaceflight. Under each such example dozens of problems requiring mathematical thought and techniques have already arisen and will continue to arise as astronautics develops new ideas and objectives. Thus the future mathematicians who will take interest in applying their art to spaceflight will have an indefinite number of new and interesting worlds to conquer.

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